TIMED GPS Attitude Determination Experiment

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ABSTRACT

This paper presents the results of an attitude determination experiment that was performed using data collected by the GPS receivers on board the TIMED (Thermosphere-Ionosphere-Mesosphere-Energetics-Dynamics) spacecraft. A novel Kalman Filter implementation was used to derive the attitude estimate, and the related dynamic and measurement equations are given. Comparisons between the GPS derived attitude estimate and the precise onboard attitude solution derived from gyros and star trackers are also presented.

INTRODUCTION

The TIMED spacecraft was built by the Johns Hopkins University Applied Physics Laboratory (JHU/APL) and launched into orbit on Dec. 7, 2001. As part of the onboard navigation system, two radiation hardened GPS receivers (one for backup) were built by JHU/APL and installed on the satellite. These receivers track the C/A codes and carrier measurements on L1, which is sufficient to derive the position and velocity estimates used by the event-based command architecture. The attitude estimate used by the spacecraft control system is generated by the onboard Attitude Interface Unit (AIU), which contains star trackers and ring-laser gyros as sensors.

The onboard GPS receivers were not designed as part of the attitude control system, and therefore each receiver has only a single antenna attached to it (see Figure 1). The measurements from these two receivers were telemetered to the ground and post-processed at APL as part of the attitude experiment. The Kalman Filter (ref. 1) processed the data to generate estimates for two of the three spacecraft orientation angles (a full, 3-D attitude estimate using only GPS measurements requires at least three antennas). The AIU measurements from the spacecraft were also put on telemetry and used to validate the GPS based attitude solution.

The intent of this experiment was to 1) verify the validity of the TIMED receiver carrier-phase measurements, and 2) provide an additional demonstration of spaceborne GPS attitude determination. GPS attitude determination has been previously demonstrated to better then one degree accuracy for antenna baseline lengths exceeding one meter (ref. 2-4).

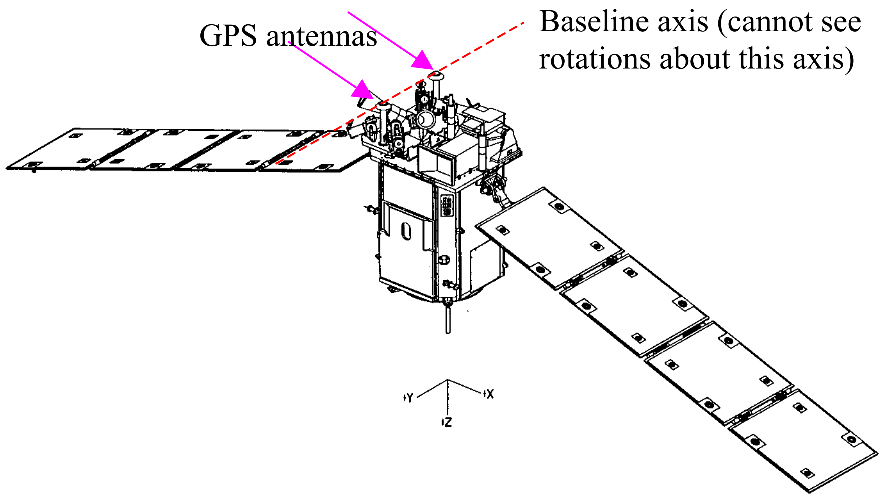

Figure 1. A diagram of the TIMED spacecraft showing the two GPS antennas used in the attitude determination experiment.

GPS OBSERVABLES

Carrier-Phase Measurement Model

The measured carrier-phase at antenna A, from GPS satellite i, is given by

$$\phi_A^i = \left\{ r_A^i + cT_A + O_A^i \right\} / \lambda_{L1} + N_A^i / k$$

ϕ_A^i = carrier phase measured by antenna A for PRN i

r_A^i = path length for antenna A / PRN i link

T_A = clock bias for antenna A

c = speed of light

O_A^i = other effects (ionosphere, multipath, linebias,etc) for antenna A / PRN i link

N_A^i = arbitrary integer

$$k = \begin{cases} 1 \text{ for message bit resolved phase} \\ 1/2 \text{ otherwise} \end{cases}$$

It is standard to process the differential carrier-phase measurements between two antennas when generating an attitude estimate. This eliminates many common mode effects. The differenced carrier-phase measurements processed by the Kalman Filter are defined by

$$
\begin{aligned}
\Delta\phi^i(t_k) &= \phi_A^i - \phi_B^i \\
&= \left\{ r_A^i(t_k) - r_B^i(t_k) + c\cdot\Delta T(t_k) + \Delta O^i(t_k) \right\} / \lambda + N^i \\
&\approx \left\{ \hat{\mathbf{e}}_i^t(t_k)\cdot\mathbf{b} + c\cdot\Delta T(t_k) \right\} / \lambda + N^i / k + \varepsilon(t_k)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\phi_{AB}^i &= \text{single difference carrier phase measurement} \\
\mathbf{b} &= \text{baseline vector connecting antennas} \\
\hat{\mathbf{e}}_i &= \text{unit vector rcvr to PRN}_\text{i} \\
\Delta T &= \text{relative clock bias} \\
N^i &= \text{relative integer} \\
\varepsilon &= \text{relative difference in other effects (primarily multipath)}
\end{aligned}
$$

Note that because the two antennas on TIMED are actually connected to two different GPS receivers (on separate oscillators) the clock bias is different for each antenna. This introduces the relative clock bias as an unknown, effectively reducing the observability of the baseline vector as compared to traditional GPS attitude systems that use multiple antennas connected to a single receiver.

In this experiment, the difference in other measurement effects (e.g. primarily multipath) was assumed to be negligible. However, the GPS antennas onboard TIMED had been calibrated prior to being mounted on the spacecraft, and this phase map (as a function of line-of-sight relative to the antenna) was used to compensate for antenna phase center variations. This compensation does not account for any spacecraft induced multipath. The assumption here is that using the mapping introduced less error then simply ignoring all line-of-sight dependent effects.

Due to the design of the receivers on board the satellite, the pseudorange was quantized to a precision that limited its utility for attitude estimation (but was sufficient to meet mission requirements), and as such it was not processed in the Kalman Filter.

KALMAN FILTER

Although the spacecraft dynamics are quite benign, a purely kinematic model was used in the Kalman Filter. This has the advantage of not requiring measurements of dynamic inputs to the spacecraft (e.g. actuator events or gyro outputs) that were not readily available. Further, it allows us to better gauge the quality of the stand-alone GPS derived attitude estimate. The inherent disadvantage of this approach is that measurements from at least four satellites must always be available to maintain a useable solution.

Dynamic Model

The state space for the model was given by

$$\text{States:}\left[\mathbf{N}, \mathbf{b}_k, \Delta T_k, L\right]$$

where,

$$\mathbf{N} = \left[N^{PRN1}, N^{PRN2}, \ldots N^{PRN12}\right] \quad \text{(relative integers)}$$

$$\mathbf{b}_k = \text{baseline vector in Earth-frame}$$

$$\Delta T_k = \text{relative clock error}$$

$$L\text{=baseline length}$$

Two comments regarding the state space are in order. First, the maximum number of GPS satellites tracked by each receiver was limited to twelve. Second, the baseline length was added to the model in order to constrain it to a constant. Although the true distance between the geometric centers of the antennas was known (it was measured prior to the launch of TIMED), the apriori covariance of the baseline length was essentially set to infinity in the filter (i.e. length treated as an unknown).

The dynamic model used in the filter was thus

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{b} \\ \Delta T \\ L \end{bmatrix}_{t_{k+1}} = \begin{bmatrix} S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{N} \\ \mathbf{b} \\ \Delta T \\ L \end{bmatrix}_{t_k} + \begin{bmatrix} \delta\mathbf{N} \\ \delta\mathbf{b} \\ \delta T \\ 0 \end{bmatrix}_{t_k}$$

$$\text{S=diag(s}_{\text{ii}})$$

$$\text{s}_{\text{ii}} = \begin{cases} 1 & \text{coherent track} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta\mathbf{N}_i = \begin{cases} 0 & \text{coherent track} \\ \sim N(0,\infty) & \text{otherwise} \end{cases}$$

$$\delta\mathbf{b}_i \sim N(0,\infty) \quad \text{(Pure Kinematic Mode)}$$

$$\delta T \sim N(0,\infty)$$

where $x \sim N(\mu, \sigma^2)$ denotes a Gaussian random variable with mean μ and variance σ^2.

The covariance of $\delta\mathbf{N}_i$ was 0 if the receivers coherently tracked the signal to the i^{th} satellite across the two measurement epochs, otherwise it was set to a large number to account for the loss of lock.

GPS Measurement Model

The GPS measurement model used in the filter was

$\mathbf{y}_k \equiv \left[\Delta\phi^1 \quad \Delta\phi^2 \quad \bullet \quad \bullet \quad \Delta\phi^{12} \right]^t - \hat{\varepsilon}_k$ stacked phase difference measurements

$\hat{\varepsilon}_k =$ carrier phase map compensation

$$\mathbf{y}_k = \left[\mathrm{I} \quad \mathbf{E}_k / \lambda \quad c/\lambda \quad 0 \right] \begin{bmatrix} \mathbf{N} \\ \mathbf{b} \\ \Delta T \\ L \end{bmatrix}_{t_k} + v_k \qquad \text{measurement model}$$

$v_k \sim N(0, \sigma^2) \qquad I.I.D.$

$\mathbf{E}_k = \left[\hat{\mathbf{e}}_1 \quad \hat{\mathbf{e}}_2 \quad \bullet \quad \bullet \quad \hat{\mathbf{e}}_{12} \right]^t$ unit vector matrix

A time sequence of carrier-phase measurements is initially required for the state to become fully observable, as shown in ref 5. The basic requirement is that $\mathbf{E} = diag(\mathbf{E}_k)$ has full column rank. Derived requirements are that $\mathbf{E}$ be "tall" (or at least square) and that no column of $\mathbf{E}_k$ be constant for all *k.*

Baseline Length PseudoMeasurements

In order to account for the baseline length constraint (because the antennas are fixed to the spacecraft), a nonlinear "pseudomeasurement" was added to the model, which was defined by

$y = |\mathbf{b}| - L$ (measurement construction)

$y = \hat{\mathbf{e}}_b^t \cdot \delta\mathbf{b} - \delta L + v$ (measurement model used in filter)

v = small noise

The noise was set arbitrarily small, which effectively "soft" constrains the baseline length to a constant. The psuedomeasurement is only processed after the carrier-phase measurement update is done, and only then if the subsequent error covariance in $\mathbf{b}$ is small. This is to insure that the Kalman gain has the correct "sign" so that the filter does not diverge.

Integer Match PseudoMeasurements

Given the model described above, the relative clock bias and relative integers are not separately observable (only their combination is observed). However, the difference in relative integers ($\Delta N_{ji} = N_j - N_{i \neq j}$) can be observed. This is essentially the double difference measurement (ref. 6) between the i^{th} and j^{th} satellites, and it removes the relative clock bias.

In order to account for the constraint that ΔN_{ji} truly be an integer, a pseudomeasurement of the form

$$y = \Delta N_{ji} = round(\Delta N_{ji}) + \text{small noise}$$

is introduced when

$$\left| \Delta N_{ji} - round(\Delta N_{ji}) \right| < p\left(\lambda / k\right) \text{ and } \sigma(\Delta N_{ji}) < p\left(\lambda / k\right) \text{ for small } p$$

This pseudomeasurement functions as a soft constraint that drives the Kalman filter estimates of ΔN_{ji}, which had been carried as "floats", to integers. The "softness" of the constraint is determined by the amplitude of the measurement noise. However, the constraint pseudomeasurement is only introduced once ΔN_{ji} is sufficiently close to an integer value, and the covariance of ΔN_{ji} is small relative to unity so that the correct constraint is likely to be applied.

The form of the Kalman Filter described above has the advantage of minimizing the bookkeeping involved over one that processes the double difference measurements directly. The loss or acquisition of any satellite only results in the "integer pseudomeasurements" being reformulated, not the underlying dynamic model and its associated covariance.

RESULTS

Two separate data sets were processed as part of this experiment. The first set consisted of 11 hours of GPS data collected on Dec 15, 2001. The second set consisted of 24 hours of GPS data collected between March 10-11, 2003. As part of these data sets, measurements from the AIU on TIMED were also available. The AIU generates an attitude solution with accuracy better than 14 arc seconds, which is far more accurate then GPS is capable of. Therefore "truth" was known for this experiment and the output of the GPS filter could be validated. Figure 2 shows a high level diagram of the comparison methodology.

Figure 3. is a plot of the error in the azimuth (defined as a rotation about the body z-axis in Figure 1) estimates from the Kalman Filter over about two orbits. The middle line in the plot is the true error, and the two outside lines are the $1-\sigma$ error bars. The mean azimuth error was less then 1 deg over the course of the entire 11 hour data set.

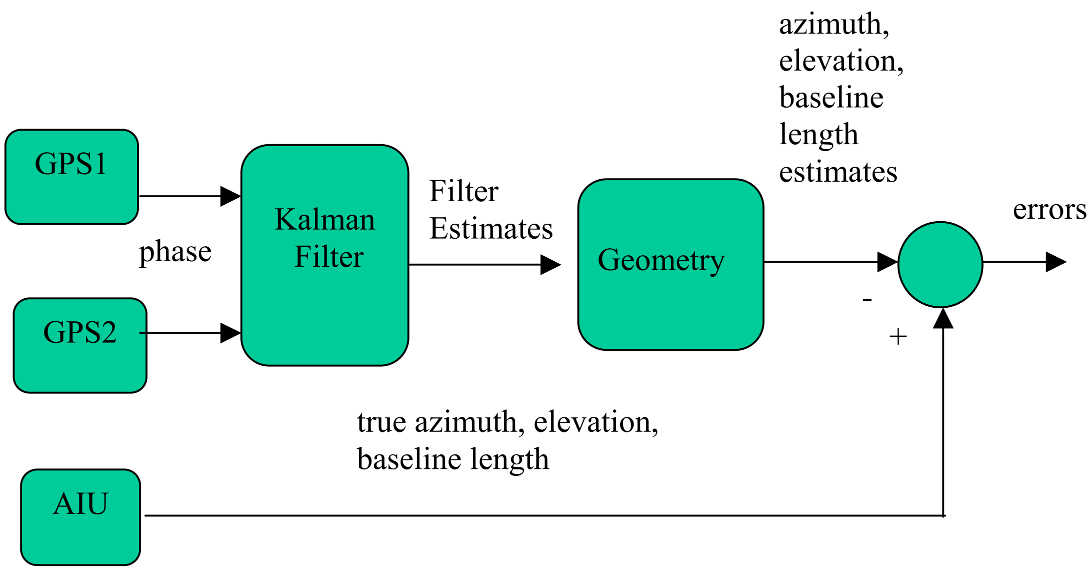

Figure 2. High-level diagram of the attitude estimate generation and validation.

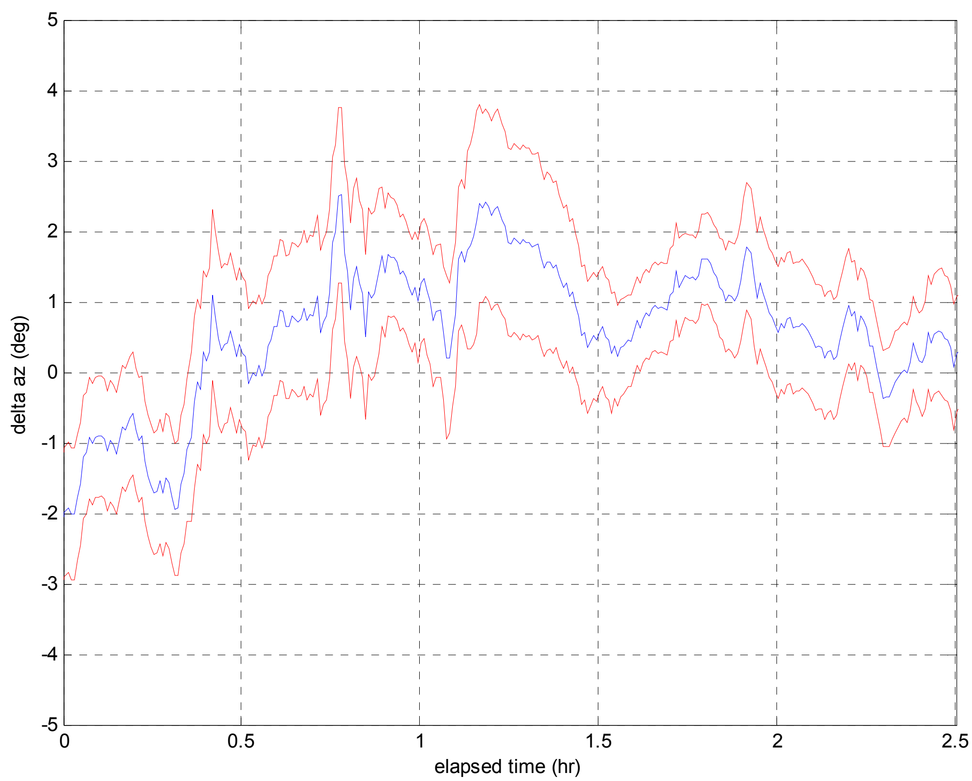

Figure 3. Error in azimuth (deg) and $1-\sigma$ error bars, for data set collected on Dec 15,2001.

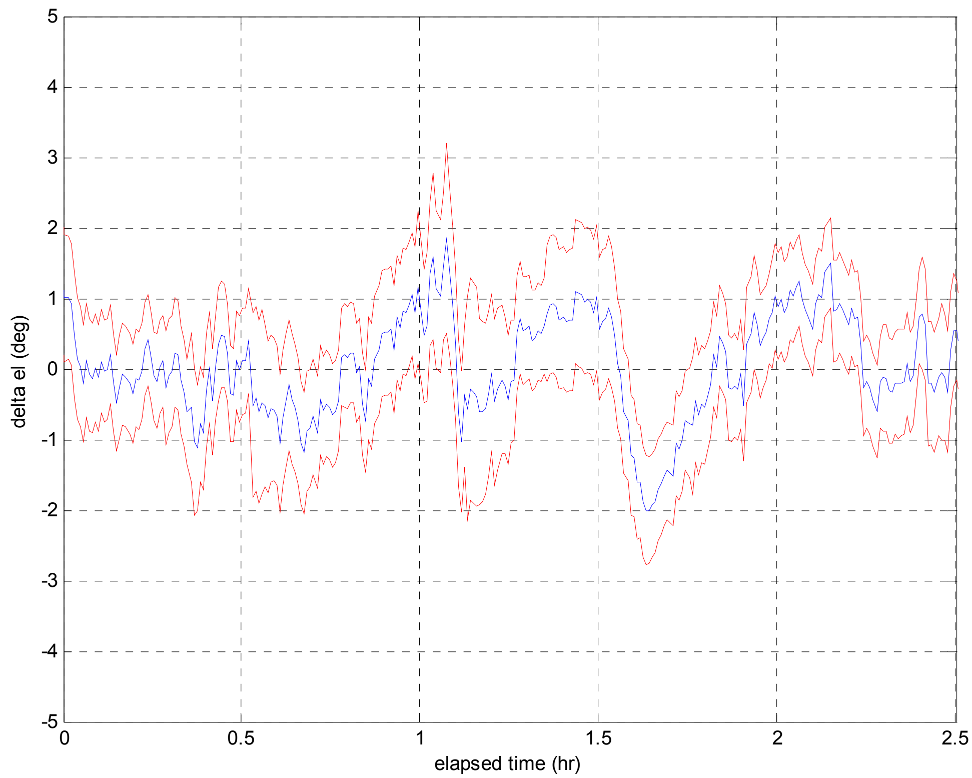

Figure 4. Error in elevation (deg) and $1-\sigma$ error bars, for data set collected on Dec 15,2001.

Figure 4. is a plot of the error in the elevation (defined as a rotation about the body x-axis in Figure 1) estimates from the Kalman Filter over the same time period as shown in figure 3. Similar to Figure 3, the middle line in the plot is the true error, and the two outside lines are the $1-\sigma$ error bars. The mean elevation error was also less than 1 degree over the entire data set.

The first data set did not have the sign of the message bit resolved (k=1/2). The second data set (collected in March, 2003) did have the message bit sense resolved (k=1), but produced similar error results.

CONCLUSIONS

This paper presented details of the GPS attitude determination experiment that was conducted using data collected from the TIMED spacecraft. A novel Kalman Filter design suitable for onboard processing which allows the baseline and integer constraints to be easily incorporated into the filter was also given. Comparison of the Kalman Filter estimates with the outputs of the AIU show that the GPS attitude estimates were accurate to better then one degree, indicating nominal operation of the TIMED GPS receiver carrier tracking loops. Higher accuracy attitude determination requires in-situ antenna and multipath calibration similar to the kind described in ref. 7.

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